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# Extending Fun with Kaprekar Number

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**ABSTRACT**

Numbers are fundamental to our living. Finding ways to explore them further takes us closer in our journey to understanding them and being amazed at simple things. At first glance, 6174 may look unassuming, but it would enthral you with its beauty, like how it did to D R Kaprekar. Computing with algorithms will extend that fun even more and explore more.



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## 1.0 INTRODUCTION

The fun ends at 6174 in the number universe of all 4-digit numbers. But the journey starts with any non-palindromic four-digit number and follows this sequence of arithmetic operations.

Rearrange the string of digits to form the highest and lowest 4-digit numbers possible

Subtract the smallest number from the largest number. Repeat the process with the steps above, and you are sure to arrive at the magic number 6174, within a maximum of 7 steps.

This simple arithmetic operation is called the Kaprekar Operation (Kaprekar, 1980), with its name derived from one of the greatest Indian mathematicians, D.R. Kaprekar, who discovered this magical number in the year 1980. Want to try the fun?

Let us start with the current year 2022.

Re-arranging the numbers, we get 2220 and 0222.

Subtracting them, we get 2220-0222 = 1998.

Repeating the process and subtracting them, we get 9981-1899 = 8082.

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Repeating again with 8082 and subtracting them, we get 8820-0288 = 8532.

Repeating with 8532 and subtracting one more time, we get 8532-2358 = 6174, the magical number.

No matter how many times, we repeat, we would stay at 6174.

Do join the fun and do this simple operation to enjoy this to the fullest. I would say begin with your birth year.

## 2.0 THE MAN BEHIND THIS FUN

Dattatreya Ramachandra Kaprekar (D R Kaprekar) was born in Dahanu, a town on the west coast of India about 100 km north of Mumbai, on 17 January 1905. Throughout his lifetime, he was fascinated by numbers and spent his whole career teaching at a high school (O'Connor and Robertson, 2007). His number theory ideas and recreational mathematics contributions were recognized by a few then, but thanks to the overall interest towards great Indian mathematicians and improvements in computational power using computers, his findings are widely recognized throughout the globe.6174, what is known as Kaprekar Constant, was first discovered by Kaprekar in 1946 and was announced at the Madras Mathematical Conference in 1949. Dr. Kaprekar published the result in the paper Problems involving reversal of digits in Scripta Mathematica in 1953 (Gupta, 2006).

## 3.0 SOLUTION USING EQUATIONS

Let us now show this Kaprekar operation through equations. Let the largest number formed by rearranging the 4 digits be represented as *abcd* and the smallest as *dcba*. Since the operation is only the reverse of the digit, we can show these numbers within the combination of these digits {a,b,c,d} itself. Taking a random 4-digit number expressed as abcd (where $a \geq b \geq c \geq d$), we can execute the Kaprekar operation as follows. The largest 4-digit number is equal to 1000a + 100b + 10c + d, so the smallest number is 1000d + 100c + 10b + a. Subtracting the smallest number from the largest and combining similar terms yields the following.

$$= 1000a + 100b + 10c + d - (1000d + 100c + 10b + a)$$

$$= 1000(a - d) + 100(b - c) + 10(c - d) + (d - a)$$

$$= 999(a - d) + 90(b - c)$$

a - d has a value between 1 and 9. b − c takes an arbitrary value between 0 and 9.

Rather than manually computing all these numbers, we can now utilize the power of programming and computational power to explore all these output numbers.

### 3.1 Coding with Programming

Below is the algorithm with the steps needed for deriving all the outputs from Kaprekar operations for all 2-digit, 3-digit and 4-digit numbers. The algorithm can be used to code a program in any programming language like Python.

#Algorithm to run Kaprekar Routine for a given number

1. Set the range of starting and ending number for the 2-digit, 3-digit, 4-digit number series.
   a) For 2-digit number series, starting number is 10 and ending number is 99
   b) For 3-digit number series, starting number is 100 and ending number is 999
   c) For 4-digit number series, starting number is 1000 and ending number is 9999

2. Iterate through the starting and ending number in a series in a for loop
3. For each number in the iteration of number series,
   a) Initialize Iteration Count as 0 to capture the number of iterations for a number to reach Kaprekar constant or 0
   b) Sort the number in descending order to get highest number
   c) Sort the number ascending order to get the lowest number
   d) Subtract the lowest number from the highest number
   e) Add the resultant to the output list to capture distinct resultants from the iterations
   f) Check if the resultant is Kaprekar constant or 0
      (i) For 2-digit number series, Kaprekar constant is 45
      (ii) For 3-digit number series, Kaprekar constant is 495
      (iii) For 4-digit number series, Kaprekar constant is 6174
   g) If the resultant is not Kaprekar constant and is not 0,
      (i) Increment the Iteration count by 1 and continue to loop through the logic
      (ii) Repeat the steps from 3a, to sort and subtract and check until Karprekar constant or 0 is returned in the resultant.
   h) If the resultant is Kaprekar constant or 0,
      (i) Add the number and iteration count to the output list to capture the # of iterations to reach Kaprekar constant for each number in the series

Continue with the next number in the number series range, step 3, until all numbers in the series are run for Kaprekar routine

### 3.2 Results with Programming

To continue the fun of Kaprekar operations, let us now explore the outputs of this algorithm.

**Output 1**

To get all the Kaprekar operations outputs after the subtraction steps. As we can see, only 55 unique numbers come as a result of the Kaprekar operations on all 4-digit numbers (1000-9999). All the distinct numbers from the iterations, except 0, sum to 9 and are divisible by 9.

Distinct values of Kaprekar numbers from iterations: [0, 999, 1089, 1998, 2088, 2178, 2997, 3087, 3177, 3267, 3996, 4086, 4176, 4266, 4356, 4995, 5085, 5175, 5265, 5355, 5445, 5994, 6084, 6174, 6264, 6354, 6444, 6534, 6993, 7083, 7173, 7263, 7353, 7443, 7533, 7623, 7992, 8082, 8172, 8262, 8352, 8442, 8532, 8622, 8712, 8991, 9081, 9171, 9261, 9351, 9441, 9531, 9621, 9711, 9801]

**Output 2**

The number of occurrences of all distinct 55 numbers from iteration on all 4-digit 9000 numbers (1000 – 9999).

0 has occurred 77 times, 999 has occurred 68 times, 1089 has occurred 78 times,
1998 has occurred 1330 times, 2088 has occurred 183 times, 2178 has occurred 45 times
2997 has occurred 214 times, 3087 has occurred 2179 times, 3177 has occurred 159 times
3267 has occurred 39 times, 3996 has occurred 2219 times, 4086 has occurred 411 times,
4176 has occurred 3370 times, 4266 has occurred 135 times, 4356 has occurred 33 times,
4995 has occurred 264 times, 5085 has occurred 453 times, 5175 has occurred 339 times,

5265 has occurred 540 times, 5355 has occurred 1090 times, 5445 has occurred 27 times, 5994 has occurred 715 times, 6084 has occurred 447 times, 6174 has occurred 8923 times, 6264 has occurred 2704 times, 6354 has occurred 1804 times, 6444 has occurred 87 times, 6534 has occurred 21 times, 6993 has occurred 218 times, 7083 has occurred 393 times, 7173 has occurred 1222 times, 7263 has occurred 261 times, 7353 has occurred 195 times, 7443 has occurred 1428 times, 7533 has occurred 63 times, 7623 has occurred 15 times, 7992 has occurred 681 times, 8082 has occurred 1697 times, 8172 has occurred 1245 times, 8262 has occurred 207 times, 8352 has occurred 3097 times, 8442 has occurred 123 times, 8532 has occurred 2099 times, 8622 has occurred 39 times, 8712 has occurred 9 times, 8991 has occurred 76 times, 9081 has occurred 141 times, 9171 has occurred 123 times, 9261 has occurred 105 times, 9351 has occurred 87 times, 9441 has occurred 69 times, 9531 has occurred 51 times, 9621 has occurred 255 times, 9711 has occurred 15 times, 9801 has occurred 3 times

**Output 3**

We see that there are around 357 numbers that reach Kaprekar number within one iteration, while there are 1980 numbers that reach Kaprekar number in the maximum 7 operations.

**Table 1 –** *Output to show the number of iterations on 4-digit numbers*

| Outcome | # Numbers | Sample numbers |
|---|---|---|
| **0 after 1 iteration** | 9 | 1111, 2222, 3333, 4444, 5555, 6666 … |
| **0 after 2 iterations** | 68 | 1000, 1011, 1101, 1110, 1121, 1211 … |
| **6174 after 1 iteration** | 357 | 1036, 1063, 1137, 1173, 1247, 1274 … |
| **6174 after 2 iterations** | 519 | 1034, 1043, 1058, 1078,1085, 1087 … |
| **6174 after 3 iterations** | 2124 | 1002, 1003, 1007, 1008, 1020 … |
| **6174 after 4 iterations** | 1124 | 1001, 1009, 1010, 1012, 1014,1017 … |
| **6174 after 5 iterations** | 1311 | 1033, 1037, 1045, 1048, 1054, 1055 … |
| **6174 after 6 iterations** | 1508 | 1013, 1015, 1016, 1018, 1031,1038 …. |
| **6174 after 7 iterations** | 1980 | 1004, 1005, 1006, 1024, 1025, 1026… |

**Output 4**

The same algorithm can be run for 2-digit and 3-digit numbers as well. And from the results, we can infer that 2-digit numbers do not converge on any constant, while 3-digit numbers all converge on 495.

## 4.0 MORE FUN AND HOW WE CAN LEARN MORE

We can now see that with the power of computing and using programming languages, the fun of numbers, as discovered by exemplary Indian mathematicians like D.R Kaprekar can be experienced at multiple levels. It would be a fascinating journey to discover these mathematical black holes in the number universe, like how a black hole in space is a place where even light cannot escape getting sucked in Wild (2018). A simple arithmetic operation like Kaprekar operation takes us into such black holes in the number universe (Sivaraman, 2021). Some of these discovered black holes are 495 for 3-digit numbers, 6174 (Kaprekar Constant) for 4-digit numbers, 549945 and 631764 for 5-digit numbers and 63317664 and 97508421 for 8-digit numbers. This journey to explore many more of these phenomena will continue to be captivating!

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