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Fascinating Algebraic Problems in *'Gaṇita Kaumudī'*

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ABSTRACT

There are many unique problems in *Gaṇita Kaumudī* (Vol. 1 GK), and this paper analyses a few of them from the 'Fractions' chapter. These types of problems on fractions are not often found in other mathematical texts. It will be a worthwhile attempt if those problems are brought to light.

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1.0 INTRODUCTION

Our ancient Indian mathematicians had a sense of humour and were fun-loving. They used to make learning and doing mathematics interesting and fun-filled activities. Interesting problems in *Gaṇita Kaumudī* from the chapter Fractions are discussed.

2.0 *BHINNAPARIKARMAN* (FRACTIONS)

'*Bhinnaparikarman*' is one of the important topics in our ancient Indian mathematical works. References can even be found in *Rg vēda, Maitrāyaṇī Saṁhitā* and *Ṡulba Sūtrā*. Indian mathematicians divided 'Fractions' into different classes (*Jāti)* according to the form.

The reduction of fractions to a common denominator is called '*Kalāsavarṇana',* meaning 'making (fractions) have the same colour'. Hindu mathematicians used to divide fractions into different classes, probably, according to the form in which these were used to be written and their nature. There is no uniformity about it (Shukla, 1971).

Nārāyaṇa Paṇḍita classifies fractions into six categories:

1) *Bhāgajāti*

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2) *Prabhāgajāti*
3) *Bhāgānubandha*
4) *Bhāgāpavāha*
5) *Svāṁśānubandha*
6) *Svāṁśāpavāha*

The need for the division of fractions into classes arose out of the lack of proper symbolism to indicate mathematical operations. The only operational symbol used by Hindus was a dot for the negative sign(Datta and Singh, 1962).

Further, *Nārāyaṇa Paṇḍita,* classified problems as equation of :

1) *Svarṇajāti*
2) *Ṣ̀eṣamūlajāti*
3) *Guṇamūla jāti*
4) *Hīnavargajāti*
5) *Amśavargajāti*
6) *Bhāgasaṃguṇyajāti*
7) *Bhinnasaṃdṛśyajāti*

This paper deals with the examples given under '*Ṣ̀eṣamūlajāti*' and a unique problem involving true or false statements.

2.1 Example 1 (GK Ex.40)

कान्तायाःसुरतप्रसङ्गसमयेभिन्नाचमुक्तावली
मुक्तानांचपदद्वयंविचरणंशय्यापटस्योपरि।
तच्छेषस्यपदंत्रिभागयुगलेनाऽऽढ्यंप्रियेणाऽऽहृतं
तच्छेषस्यपदंक्षितौनिपतितंसूत्रेद्वयंकिंवद॥

During a love quarrel, the lady's necklace made up of pearls was broken. Twice the square-root less ¼ of the root of pearls were on the cover of the bed. The square root of the rest along with 2/3 of this root were seized by the lover. The square-root of the rest fell down on the earth and 2 pearls were in the string of the garland. How many pearls were there in the garland?

Total number of pearls = x

Pearls on the bed = $2\sqrt{x} - \frac{1}{4}\sqrt{x} = \frac{7}{4}\sqrt{x}$

Balance = $x - \frac{7}{4}\sqrt{x}$ say 'a'

Seized by the lover = $\sqrt{a} + \frac{2}{3}\sqrt{a}$

Balance in the string = $a - \sqrt{a} - \frac{2}{3}\sqrt{a}$

= $a - \frac{5}{3}\sqrt{a}$ Let it be 'b'

Fallen on the floor = $\sqrt{b}$

Balance = $b - \sqrt{b}$

Balance in the string = 2

Therefore $b - \sqrt{b} = 2$

i.e., $b = 4$

->$a - \frac{5}{3}\sqrt{a}$ = 4

->$3a - 5\sqrt{a}$ = 12 solving this we get

a = 3^2 = 9

->$x - \frac{7}{4}\sqrt{x}$ = 9 ->$4x - 7\sqrt{x}$ = 36 solving this we get

-> $x = 4^2 = 16$

Total number of pearls in the garland is 16.

2.2 Example 2 (GK Ex. 41)

गणेशंपद्मेनत्रिनयनहरिब्रह्मदिनपान्
विलोमैःशेषांशैर्विषयलवपूर्वैश्चकमलाम्।
पदेनाSSपूज्यैकेनचगुरुपदाम्भोजयुगलं
सरोजेनाSSचक्ष्वद्रुतमखिलमम्भोजनिचयम्॥

Ganesa was worshipped with 1 lotus flower. Siva, Hari, Brahma and the Sun were worshipped with 1/5, ¼,1/3 and ½ of what remained, successively. Kamala was worshipped with the square-root of the lotus flowers and two feet of the teacher resembling lotus flowers were worshipped with 1 lotus flower. How many lotus flowers were there in the collection?

Total Flowers = x

Ganesa = 1

Balance = $(x-1)$

Siva = $\frac{1}{5}(x-1)$

New Balance = $\frac{4}{5}(x-1)$

Vishnu = $\frac{1}{4}(\frac{4}{5}(x-1))$

= $\frac{1}{5}(x-1)$

New Balance = $\frac{4}{5}(x-1) - \frac{1}{5}(x-1)$

= $\frac{3}{5}(x-1)$

Brahma = $\frac{1}{3}(\frac{3}{5}(x-1))$

= $\frac{1}{5}(x-1)$

New Balance = $\frac{3}{5}(x-1) - \frac{1}{5}(x-1)$

= $\frac{2}{5}(x-1)$

Sun = $\frac{1}{2}(\frac{2}{5}(x-1))$

= $\frac{1}{5}(x-1)$

Balance $= \frac{2}{5}\ (x-1) - \frac{1}{5}\ (x-1)$

$= \frac{1}{5}\ (x-1)$

Kamala $= \sqrt{x}$

Teacher $= 1$

$\frac{1}{5}\ (x-1)\ - \sqrt{x}\ - 1\ = 0$ simplifying this

$$x - 1 - 5\sqrt{x} - 5 = 0$$

$$x - 5\sqrt{x} \ = 6$$

Solving this $\sqrt{x} = 6$

$x^2 \ = \ 6^{\ 2} \ = 36$

Solution: Total number of flowers is 36.

2.3 Example 3 (GK Ex. 39)

यातेनृपेमृगयुभिर्मृगयार्थमाशु
पाशान्प्रसारयतितत्त्रिलवोऽप्यटव्याम्।
शेषस्यघोरतरकेसरीपीडितानि
त्रीणिप्रचक्ष्वसचतुष्कपदानिविद्वन्॥

A king went out for hunting. The hunters laid the trap. There, 1/3 of the deer netted themselves. Thrice the square-root of the rest along with 4 were trouble by the ferocious lion. How many of them were there?

Trapped Deer : $\frac{x}{3}$

Balance : $x - \ \frac{x}{3}$

Deer threatened by lion : $3 \ \text{x}\sqrt{\frac{2}{3}\ x} + \ 4$

Therefore

$$x = \frac{x}{3} + 3\sqrt{\frac{2}{3}x} + 4$$

$$\frac{2}{3}x - 3\sqrt{\frac{2}{3}x} = 4$$

Let y $= \frac{2}{3}\ x$

$$y - 3\sqrt{y} = 4, solving\ this\ equation$$

$$\sqrt{y} = 4$$

$$y = 4^2$$

$$y = 16$$

Since $y = \frac{2}{3}\ x, \ \ x = \ 24.$

Solution: Total Deer = 24.

3.0 TRUE OR FALSE STATEMENTS PROBLEM (GK RULE 42(B) -43 (A))

Nārāyaṇa Paṇḍita in Chapter II of *Gaṇita Kaumudī* discusses miscellaneous problems such as Partnership, Interest calculation,Gems, meeting of travellers etc. One of such problems is 'True or False'statements (Gaṇita Kaumudī, 1936; Gaṇita Kaumudī, 1998).

सत्यानृतेसूत्रम्।

सैकेष्टगुणाःपुरुषाद्विगुणेष्टानाभवन्त्यसत्यानि।

तैरूनापुरुषकृतिःशेषंसत्यानिवचनानि॥

Add 1 to (the number of) persons liked (by a lady). Multiply (the sum) by the number of persons. (The Product) less twice (the number of) persons liked is the number of false statements. Subtract the same from the square of the number of persons. The remainder is the number of true statements.

Let the lady like n persons out of m persons while others are disliked by her. Suppose that she speaks to each person that only he is liked, while others disliked.

Then the number of false statements = $m(n+1)-2n$, and

The number of True statements = $m^2 - m(n+1) + 2n$.

This can be presented as a table :

	A1	A2	A3	....	An	An+1	An+2	An+3	....	Am	TRUE	FALSE
A1	T	F	F	F	F	T	T	T	T	T	m-n+1	n-1
A2	F	T	F	F	F	T	T	T	T	T	m-n+1	n-1
A3	F	F	T	F	F	T	T	T	T	T	m-n+1	n-1
....	F	F	F	T	F	T	T	T	T	T	m-n+1	n-1
An	F	F	F	F	T	T	T	T	T	T	m-n+1	n-1
An+1	F	F	F	F	F	F	T	T	T	T	m-n-1	n+1
An+2	F	F	F	F	F	T	F	T	T	T	m-n-1	n+1
An+3	F	F	F	F	F	T	T	F	T	T	m-n-1	n+1
....	F	F	F	F	F	T	T	T	F	T	m-n-1	n+1
Am	F	F	F	F	F	T	T	T	T	F	m-n-1	n+1

TRUE $n(m-n+1)+(m-n)(m-n-1)$

$m^2 - m(n+1) + 2n$

FALSE $n\,(n-1) + (m-n)\,(n+1)$

$m(n+1) - 2n$

Hence proved.

This rule has already been given in *Gaṇitasārasaṃgraha* and *Mahāvīrācārya* classifies it under *Vicitra Kuṭṭīkāra* problems (curious and interesting problems involving proportionate division). The rule and the example given by GK and GSS read almost verbatim.

4.0 CONCLUSION

Classification of fractions by various ancient Indian mathematical texts alone can be taken up as a separate study to understand the continuity of computational tradition of our ancient wisdom.

There are many such unique problems in the ancient Indian mathematical texts. Introducing the present generation of students to these types of problems will allay their fears about the complexity of the ancient works.

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