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Mathematical Ideas in Bakhshali Manuscript

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ABSTRACT

The Bakhshali manuscript is written on birch bark and according to its carbon-date proposed it was to be written at AD 224-383/ 885-993 and discovered in 1881 at Bakhshali village, Mardan – Peshawar at the present day it belongs to Pakistan, in the ancient day it belongs to India - on side of Indo-Iran. It is written in a form of literary Sanskrit. This manuscript comprises mathematical text that focuses on contents like the earliest use of zero symbol, Numerals, Arithmetic, Algebra, Geometry, Fractions & Square roots, Progression, Profit, Loss & Interest, Quadratic Equations, Intermediate Equations & many more. The first research on this Manuscript was done by Augustus Frederic Rudolf Hoernle after his death, the manuscript was further examined by G. R. Kaye who edited and published the work as a book in 1927. Some of the contents in this manuscript like the zero symbol, Numerals, Fractions, etc. are compiled from published books and other sources are studied & enlisted in this paper. The main point of focus is on earlier records of zero, a brief note on construction in mathematical formulation, Shunya-Bindu, Fractions, and facts that Indians far before invented various concepts in mathematics than the known timeline.

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1.0 INTRODUCTION

The origin of the Bakshali manuscript and how it was excavated from the village of Bakhshali and further discussion about the timeline of the discovery is compiled and enlisted.

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1.1 Origin & Discovery

One of the ancient mathematical texts written on birch bark was discovered in 1881 in the village of Bakhshali, Mardan in the ancient day, it belonged to India – on the side of Indo-Iran, at Present day near Peshawar - Pakistan. It is possibly known as "the oldest extant manuscript in Indian mathematics". For some portions of the manuscript, the carbon–date was proposed as AD 224-383, while in the other portion, the carbon–date was proposed as AD 885-993 in recent studies. The format is the Bakhshali manuscript currently comprises 70 leaves (like 70 pages). The Sharada script is followed in the writing of the manuscript. The Sharada script belongs to the Sanskrit language. The content is maths text; currently, the manuscript is too fragile to handle.

Other studies also say that Bakhshali village is geographically situated near Pushkalavati and Takshasila - As per Valmiki Ramayanam, sons of Bharata - Rama's brother. They defended against the Gandharan Tribes and eventually established two cities in that region, and the cities were named after their names Taksha & Pushkala. And as per studies, the world's first University was established in Takshasila in 700 BC, and researchers also have the point of view that maybe the Bhakshali manuscript can be written by scholars from Takshasila University.

Bakhshali is majorly centered in the Indo-Greek-Roman confluence as the word Dinara (Dinarius - Currency of ancient Rome) & Dramha (Drachma – Currency of ancient Greek) denominations of money used in profit & loss calculations.

1.2 Timeline

Inspector of Police named Mian An-Wan-Udin, whose tenant actually discovered the manuscript while digging a stone enclosure in a ruined place, took the work to Assistant Commissioner at Mardan who intended and forwarded the manuscript to Lahore Museum, it was subsequently sent to Lieutenant Governor of Punjab, On the advice of General A. Cunningham, it was passed on to Dr. Rudolf of the Calcutta Madrasa for study and publication. He described the Bakhshali manuscript before the Asiatic Society of Bengal in 1882, which was published in the Indian Antiquary in 1883. He gave a detailed account of his work at the Seventh Oriental Conference held in Vienna in 1886. And a revised version of this paper is in the Indian Antiquary 1888. In 1902, he presented the Bakhshali manuscript to the Bodleian Library, Oxford, and it is still there. And then after Dr. Rudolf many others studied the manuscript, G. R. Kaye was the notable one among them.

2.0 THE SCRIPT AND WRITING STYLE

The Sharada Script is used in the writing of this Bakhshali manuscript, and the writing style follows a SUTRA for it which are further discussed in this topic.

2.1 Sharada Script

The Sharada Script, named after Saraswathi Devi is an ancient writing system of Samskrutam & an ancestor of Gurmukhi (Punjabi). This Sharada Script is almost extinct as we are used to Devanagiri Script. Kashmiri pandits use Sharada Script for their writings. The variant of the Sharada Script is Siddhamatrika popular in the Northwest Region of India 1500 – 1800 years ago.

Brahmi → Sharada → Siddhamatrika → Devanagiri

The Bakhshali manuscript is written in Sharada Script and has the following Sutra,

The SUTRA-s of Bakhshali Manuscript:

Minimum Alphabets, Unambiguous, Concise, Universal, Non-Redundant, without a flaw, these are the SUTRA-s followed. This Sutra method is also followed by many Rishis to briefly articulate the knowledge.

3.0 THE CONTENTS

The dot symbol represents Shunya-Bindu which was earlier in use by this manuscript before Aryabhatta invented the digit zero, and between the 2nd and 3rd centuries how ancient Indians had thoughts in Mathematical Formulation are briefly listed.

3.1 Shunya-Bindu & Mathematical Formulation

Figure 1 – *Numerals from the Bakhshali manuscript, which was discovered by Augustus Hoernle in 1887 and believed to have been written between the 2nd century BC and 3rd century AD*

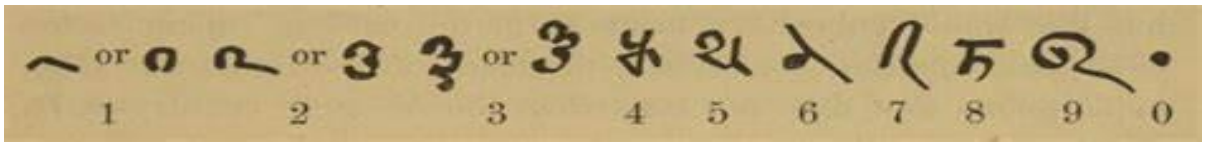

Figure 2 - *Use of Shunya-Bindu in Bakshali Manuscript*

The manuscript comprises rules and illustrative examples, the manuscript uses numerals with a placeholder for zero (*see* Figure 1) "*Numerals from the Bakhshali manuscript, which was discovered by Augustus Hoernle in 1887 and believed to have been written between the 2nd century BC and 3rd century AD".* The dot symbol is called the Shunya-Bindu, which is literally, the dot of the empty place.

Scientists have also traced the earliest record of zero in this Bakhshali manuscript (*see* Figure 2).

Bakhshali manuscript focuses on mathematical texts; the contents of the manuscript are Arithmetic, Algebra, Geometry, Fractions & Square Roots, Progressions, Profit, Loss & Interest, Quadratic Equations, Indeterminate Equations, etc. As G. R. Kaye says in his book, the solution for the Indeterminate Equations of the first degree was found in this manuscript before Euler and Lagrange. The construction in which the mathematical formulation is articulated:

SUTRAM → TADA → STHAPANAM → KARANAM → PRATYAYAM

SUTRAM (The Rule)

TADA (The Example in words)

STAPANAM (The Example in numbers)

KARANAM (The Solution – idea)

PRATYAYAM (The Proof)

This was further discussed in Dr. Rudolf book about the Bhakshali manuscript.

4.0 FRACTIONS

Fractional notation in the Bakhshali manuscript is dissimilar to what we write today, which is writing one number below the other, for example -$\frac{1}{2}$

Figure 3 - *Fractions Written on the Birch Bark Leaves in the Bakhshali Manuscript*

The addition symbol we use nowadays + is here used as the symbol for subtraction. Dr. Rudolf translates Sanskrit to English, and thus,

3/4 – 1/2 is written as	3 1+ 4 2 means 3/4 minus 1/2
1+1/3 is written as	3 1 1 3 means 1 plus 1/3 so equals 4/3
1-1/3 is written in the following way	1 1 3+ means 1 minus 1/3 so equals 2/3
Sums of the fraction 5/1+2/1 are written with the symbol yu (for yuta)	5 2 yu 1 1 pha 7 This means 5/1 plus 2/1 equals 7
The Division is denoted by Bha (Bhaga - part)	1 1 bha 8 3+ pha 12 This means 8 divided by 2/3 equals 12
Phala means equal to	● 1 1 1 1 1 1 1 bha 32 3+ 3+ 3+ phala 108 Find number 32 divided by $(2/3)^3$. It equals 108

The equation given with the large dots are representing unknowns, A confusing aspect is that it is also used to represent zero.

Other concepts like square roots are also studied via this manuscript: In the case of a non-square number, subtract the nearest square number, divide the remainder by twice this nearest square; half the square of this is divided (/) by the sum (+) of the approximate root and the fraction. This is subtracted and will give the corrected(true) root. – translated from the original manuscript. The result from the Bakhshali manuscript is correct up to five decimal places.

Example: $\sqrt{105}$ = 10.24695122 (Bakhshali manuscript), $\sqrt{105}$ = 10.24695077 (calculator approx.)

5.0 CONCLUSION

Bhakshali manuscript serves as evidence for the planting of mathematical ideas in the ancient period that is before 1800 years, and also with these 70 birch bark leaves of the manuscript, we get this much information related to many topics in mathematics, and some are under study in Bodleian Library, University of Oxford. And from this paper, we get to know that Ancient Indians had thought about various concepts in mathematics before 1800 years. And the concepts of the Shunya-Bindu and Fractions & Square roots presently create awe in our minds.

REFERENCES

Dr. Rudolf's paper - Indian Antiquary 1888.

https://en.wikipedia.org/wiki/Bakhshali

https://mathshistory.st-andrews.ac.uk/HistTopics/Bakhshali_manuscript/

https://tbhakshalimt.blogspot.com

https://youtu.be/Ufl_iS6nniU - The Bakhshali Manuscript