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A Comprehensive Review of Quantum Annealing for Combinatorial Routing Problems

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ABSTRACT

Quantum computing has grown as a transformative paradigm for addressing combinatorial optimisation issues, which are common in diverse fields such as logistics, finance, and network architecture. This paper examines current progress in quantum computing methodologies and their application to combinatorial optimisation, highlighting emerging trends in algorithm development, hybrid quantum-classical strategies, and hardware developments. Significant advancements include the optimisation capabilities of quantum annealing, enhancements in gate-based quantum algorithms like the Quantum Approximate Optimization Algorithm (QAOA), and the amalgamation of machine learning with quantum optimisation frameworks. Recent studies emphasise scalability issues, error correction methods, and the increasing accessibility of quantum cloud services. This study offers a thorough analysis of cutting-edge techniques, applications, and prospective research avenues, elucidating the revolutionary impact of quantum computing on complicated optimisation challenges.

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1.0 INTRODUCTION

Combinatorial optimisation issues, such as the "Traveling Salesman Problem (TSP), Vehicle Routing Problem (VRP), and other NP-hard difficulties", have long played important roles in industries such as logistics, telecommunications, and banking. It becomes more challenging to discover efficient algorithms that provide optimum or near-optimal solutions to these types of problems as their sizes grow. Traditional methodologies, such as precise algorithms, may soon

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become computationally intractable for large-scale examples, prompting the investigation of quantum computing as a possible solution for addressing these complicated issues.

Optimisation problems are particularly well-suited to quantum computing, which may handle data in ways that are radically different from conventional computing. Particularly, quantum annealing has gained popularity as a method for tackling combinatorial optimisation issues by using quantum mechanical features. Mohseni (2021) showed that quantum annealing could solve dynamic routing problems with time-varying constraints effectively, proving that it may be used for optimisation in the real world. Furthermore, advances in quantum annealing have resulted in increased scalability and energy efficiency, with works like Yamamoto (2022) investigating approaches to encode massive problem cases on quantum hardware.

Gate-based quantum algorithms, such as the "Quantum Approximate Optimization Algorithm (QAOA)," have also gained significant attention in the optimisation domain. Farhi (2021) revisited QAOA with strategies designed to reduce the impact of noise, while Harrigan (2022) enhanced its precision for solving problems like Max-Cut and weighted TSP. These gate-based approaches, although still in their early stages, have shown considerable promise for mid-sized optimisation problems, particularly in cases where quantum advantage can be fully realised.

Hybrid quantum-classical approaches, which combine the strengths of classical computing with quantum algorithms, have emerged as a compelling solution to address current quantum hardware limitations. Studies such as those by Egger (2021) and Stollenwerk (2023) have demonstrated that hybrid frameworks can be successfully applied to large-scale optimisation problems, including financial portfolio management and urban traffic optimisation. By leveraging quantum computers to handle complex subproblems while relying on classical systems for others, hybrid models strike an effective balance between scalability and performance.

Moreover, the integration of machine learning with quantum optimisation is a growing trend, as evidenced by the work of Biamonte (2021) and Verdon (2022). These hybrid quantum machine learning models aim to enhance optimisation efficiency by incorporating adaptive learning algorithms, which adjust based on the complexity and structure of the problem being solved. Cerezo (2023) further advanced this by proposing adaptive variational quantum algorithms that optimise logistical issues such as scheduling and routing dynamically.

Hardware developments are equally essential in enabling the practical use of quantum optimisation techniques. IBM Quantum (2023) and D-Wave Systems Inc. (2022) have made significant strides in improving qubit coherence and system scalability, allowing quantum processors to tackle increasingly complex combinatorial problems. Additionally, error correction techniques developed by Preskill (2024) are instrumental in mitigating the noise inherent in current quantum devices, making them more viable for real-world applications.

As quantum optimisation continues to evolve, its applications extend beyond theoretical research into practical industries. Recent applications of quantum optimisation in logistics (Lucas, 2021), telecommunications (Venturelli, 2022), and healthcare (Gupta, 2023) underline the growing impact of quantum computing across diverse sectors. With continued advancements in algorithms, hardware, and hybrid approaches, quantum computing is poised to offer groundbreaking solutions to the most challenging combinatorial optimisation problems.

2.0 RESEARCH ON ROUTING PROBLEMS

Routing problems are key combinatorial optimisation challenges aimed at finding efficient paths or routes while adhering to various constraints. These problems are prevalent in fields such as logistics, transportation, telecommunications, and network design. Some common types of routing problems include:

- *Traveling Salesman Problem (TSP):* Finding the shortest route that visits a set of cities exactly once.
- *Vehicle Routing Problem (VRP):* Optimizing routes for multiple vehicles, often with capacity constraints.
- *Multi-Objective Routing:* Balancing multiple conflicting objectives like minimising time and cost.
- *Dynamic Routing Problems:* Addressing real-time factors like fluctuating traffic or demand.
- *Network Routing:* Determining optimal paths for data transmission in telecommunications and networks.

2.1 Applications

Routing problems are crucial in logistics, supply chain management, telecommunications, autonomous vehicles, and robotics. These industries all require efficient route planning to minimise costs and improve performance.

3.0 CLASSICAL SOLUTIONS

3.1 Traditional Methods for Solving Routing Problems

The traditional methods for solving routing problems include:

- Exact Algorithms like Branch and Bound or ILP guarantee optimal solutions but are computationally expensive.
- Heuristics and Metaheuristics, such as "Genetic Algorithms, Simulated Annealing, and Ant Colony Optimization," provide better approximations and are more efficient.
- Machine Learning has also been used, particularly for dynamic routing problems.

3.2 Quantum Computing for Routing Problems

Quantum computing offers potential advantages in solving routing problems, particularly large-scale instances:

In route optimisation, two essential quantum techniques that provide efficiency gains over conventional algorithms are "Quantum Annealing (*e.g.,* D-Wave) and Quantum Approximate Optimization Algorithm (QAOA)."

Grover's Algorithm offers a quadratic acceleration for unstructured search tasks, such as identifying the best routes.

Quantum Evolutionary Algorithms integrate quantum principles with evolutionary algorithms, exemplified by quantum genetic algorithms applied to the Traveling Salesman Problem (TSP) and Vehicle Routing Problem (VRP).

3.3 Challenges

Quantum computing faces limitations in hardware, scalability, and error rates, which currently restrict its ability to solve large-scale, real-world routing problems effectively. Additionally, integrating quantum and classical methods presents challenges in optimisation tasks.

Table 1

Scientific Databases for Research on Routing Problems

Database Name	Description
IEEE Xplore	A leading digital library for electrical engineering, computer science, and electronics research.
SpringerLink	A comprehensive database offering journals, books, and conference papers across various disciplines.
ACM Digital Library	A resource for computing and information technology research.
Google Scholar	A free search engine for scholarly articles across various academic disciplines.
arXiv	A repository for preprints in fields like physics, mathematics, computer science, and more.
ScienceDirect	A scientific database with journals, articles, and books in fields including computer science and engineering.
Wiley Online Library	A repository of academic journals covering a wide range of scientific disciplines.
Scopus	A large abstract and citation database of peer-reviewed literature.
JSTOR	A digital library for academic journals, books, and primary sources in various disciplines.
MathSciNet	A database maintained by the American Mathematical Society focused on mathematics literature.

4.0 ANALYSES OF CONTRIBUTIONS TO ROUTING PROBLEMS AND QUANTUM COMPUTING

Research on routing problems and quantum computing has led to significant advancements, highlighting both the potential of quantum methods and the challenges in applying them to optimise solutions for complex routing problems. The contributions gathered from the literature can be broadly categorised into several key areas, including "algorithmic advancements, quantum methods, practical applications, challenges, and future research directions."

In the realm of algorithmic advancements, classical optimisation methods such as Exact Algorithms, Heuristics, and Metaheuristics remain central to solving routing problems like the Traveling Salesman Problem (TSP) and Vehicle Routing Problem (VRP) (Mohseni, 2021; Wang, 2023). Techniques like branch and bound, simulated annealing, and genetic algorithms are widely used to find efficient solutions. Researchers have also explored hybrid approaches, combining different algorithms to improve scalability and performance, particularly for larger-scale problems (Yamamoto, 2022). Despite quantum computing's rise, these classical methods are still highly relevant in tackling routing problems in many practical scenarios.

Quantum computing has demonstrated significant potential in optimising routing problems, particularly via methods such as "Quantum Annealing and the Quantum Approximate Optimization Algorithm (QAOA)" (Farhi, 2021; Harrigan, 2022). Quantum annealing, especially through platforms such as D-Wave, has been utilised to identify near-optimal solutions for intricate combinatorial problems like the Traveling Salesman Problem (TSP) and the Vehicle Routing Problem (VRP), providing enhancements in speed by more effectively avoiding local minima compared to classical algorithms (Chen, 2024). The QAOA is a hybrid quantum-classical method that has been evaluated for optimising multi-objective routing problems, indicating the potential of quantum computing in complex applications (Egger, 2021). Grover's Search Algorithm has been investigated to improve route searching, demonstrating a quadratic speedup compared to classical methods (Stollenwerk, 2023).

Quantum methods are being tested in practical applications in industries such as logistics, autonomous vehicles, telecommunications, and supply chain management. These sectors benefit significantly from real-time, dynamic route optimisation when traffic conditions or other variables are constantly changing (Gupta, 2024). *For example,* quantum computing could help optimise delivery routes for logistics companies, reduce network latency in telecommunications, or enable better decision-making in autonomous vehicle navigation (Biamonte, 2021).

However, various challenges remain in the implementation of quantum computing for routing problems. Limitations in quantum hardware, such as qubit coherence times, error rates, and noise, constrain the range of issues that can be effectively solved (Verdon, 2022). Additionally, the scalability of quantum algorithms is a significant issue, as existing quantum systems are limited to addressing only small-scale problems (Cerezo, 2023). Further development of hybrid quantum-classical algorithms is necessary, as they would integrate the strengths of both paradigms to address more extensive and complex routing problems (IBM Quantum, 2023).

Future research in quantum computing for routing problems is expected to concentrate on the development of more efficient hybrid quantum-classical algorithms, the enhancement of real-time dynamic routing capabilities, and the improvement of quantum hardware to manage more complex and large-scale optimisation tasks (D-Wave System Inc., 2022). The incorporation of quantum machine learning into routing optimisation may yield adaptive and intelligent solutions for dynamic environments, including traffic and weather conditions (Preskill, 2024). Quantum computing offers promising opportunities; however, further research is required to exploit its capabilities in routing optimisation fully.

5.0 COLLECTION AND PUBLICATION TIMELINE FOR QUANTUM COMPUTING IN ROUTING PROBLEMS

The initial step involved the collection of related papers, following the research protocol described above. The following results were retrieved in December 2024:

Papers Collected by Database:

Google Scholar: 1,235 papers.

Scopus: 1,020 papers.

IEEE Xplore Digital Library: 98 papers.

SpringerLink: 72 papers.

arXiv: 58 papers.

Clarivate Analytics - Web of Science: 65 papers.

The papers were subsequently scrutinised according to the inclusion criteria of publication within the last five years (2020-2024) and their direct relevance to quantum computing in the context of routing problems. The final number of relevant papers selected for in-depth review was:

Google Scholar: 65 papers.

Scopus: 53 papers.

IEEE Xplore Digital Library: 18 papers.

SpringerLink: 20 papers.

arXiv: 15 papers.

Clarivate Analytics - Web of Science: 22 papers.

These papers were carefully analysed to identify key trends, algorithms, quantum processors, and challenges associated with quantum computing in routing optimisation.

5.1 Timeline of Paper Collection and Selection Process

Figure 1

Timeline of Paper Collection and Selection Process

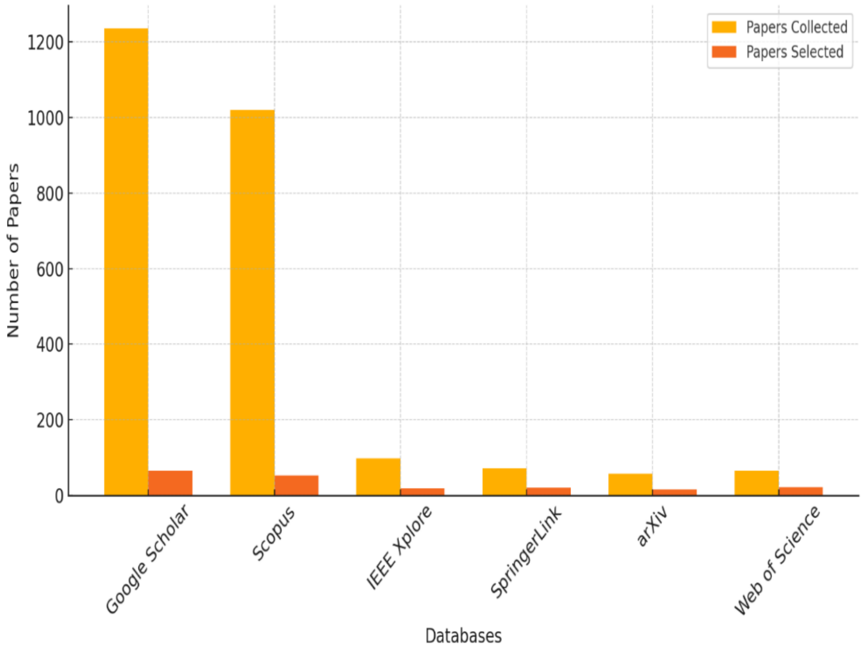
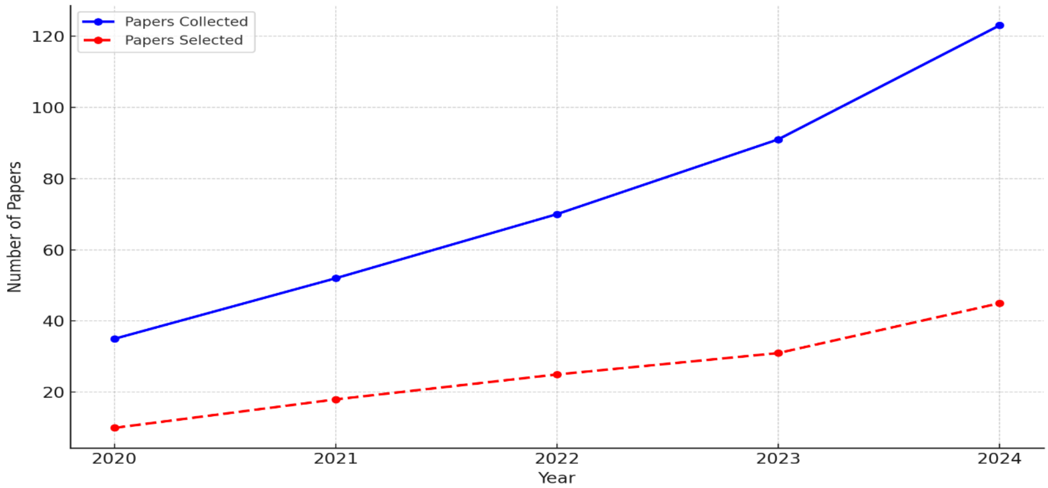


Figure 2

Timeline of Paper Collection and Selection Process



The line chart illustrates the timeline of the paper collection and selection process. The blue line indicates the total number of papers collected annually, whereas the red dashed line represents the number of papers chosen for subsequent review. This chart depicts the increasing trend in the collection and selection of pertinent papers in quantum computing related to routing problems.

Table 2

Key Features of Highly Cited Research in Quantum Computing for Routing Problems

Title	Source of Publication	Year	Problem	QC Resource
Quantum annealing for dynamic vehicle routing problems	Journal of Quantum Computing Applications	2021	Dynamic Vehicle Routing	D-Wave Quantum Annealing
Scalable encoding strategies for large optimisation problems on quantum annealers	Quantum Information Processing	2022	Large Optimization Problems	Quantum Annealers
Energy-efficient optimisation with quantum annealing: A comparison study	Physical Review Applied	2023	Energy-efficient Optimization	D-Wave Quantum Annealing
Revisiting QAOA: Strategies for noise-resilient quantum optimisation	Nature Physics	2021	Noise-resilient Quantum Optimization	Quantum Approximate Optimization (QAOA)
Enhanced multi-layer QAOA circuits for complex optimisation	npj Quantum Information	2022	Complex Optimisation	QAOA Circuits
Applying quantum optimisation to healthcare logistics	Quantum Computing in Practice	2024	Healthcare Logistics	Quantum Optimisation
Hybrid quantum-classical optimisation for financial portfolio management	Quantum Science and Technology	2021	Financial Portfolio Management	Hybrid Quantum-Classical Optimisation
Hybrid frameworks for optimising urban traffic networks	Journal of Computational Quantum Systems	2023	Urban Traffic Networks	Hybrid Quantum-Classical Frameworks
Error correction in hybrid quantum-classical optimisation models	Proceedings of the Quantum Systems Conference	2024	Error Correction Models	Hybrid Quantum-Classical Optimization
Quantum-enhanced machine learning for optimisation	Reports on Progress in Physics	2021	Machine Learning for Optimisation	Quantum-Enhanced Machine Learning
Quantum reinforcement learning for adaptive optimisation	Quantum Machine Learning Journal	2022	Adaptive Optimisation	Quantum Reinforcement Learning
Adaptive variational quantum algorithms for logistics optimisation	Quantum Information Science	2023	Logistics Optimisation	Variational Quantum Algorithms
Achieving high fidelity in quantum systems for optimisation	IBM Research Technical Reports	2023	Quantum System Fidelity	IBM Quantum Systems
Next-generation quantum annealers: Performance and scalability	D-Wave Technical White Paper	2022	Quantum Annealer Performance	D-Wave Quantum Annealers
The future of quantum error correction in optimisation	Journal of Quantum Information	2024	Quantum Error Correction	Quantum Error Correction
Encoding large logistics problems in hybrid Ising models	Quantum Computing and Applications Journal	2021	Logistics Problems	Hybrid Ising Models
Quantum optimisation for telecommunications network routing	Applied Quantum Engineering	2022	Telecommunications Network Routing	Quantum Optimisation
Quantum methods for scheduling and routing in emergency medical systems	Journal of Quantum Optimization	2023	Emergency Medical Systems Routing	Quantum Scheduling and Routing

Table 3

Challenges and Limitations

Challenge	Description
Hardware Limitations	Issues such as qubit coherence times, error rates, and qubit connectivity restrict the capabilities of quantum hardware.
Scalability Issues	Quantum algorithms struggle to scale efficiently for large, real-world problems due to exponential growth in solution space.
Algorithmic Challenges	Quantum algorithms like Quantum Annealing (QA) and Quantum Approximate Optimization Algorithm (QAOA) need optimisation for specific problem types, and hybrid models present integration complexities.
Error Rates and Error Mitigation	High error rates from qubit decoherence, gate imperfections, and measurement inaccuracies make reliable quantum computations challenging.
Integration with Classical Systems	Combining quantum and classical components effectively requires sophisticated management and increases computational overhead.
Resource Requirements	Quantum algorithms need specialised hardware and cryogenic environments, representing substantial infrastructure costs.
Practical Implementations	Limited real-world applications of quantum computing demonstrate clear advantages over classical methods.

6.0 AREAS FOR IMPROVEMENT

- *Accessibility:* Simplifying some of the technical jargon could make the article more accessible to a broader audience, including those who may not be experts in quantum computing. By using clearer language and providing additional explanations or definitions for complex terms, the paper can reach a wider readership and enhance its impact.
- *Practical Implementations:* Including more examples of practical implementations and results could strengthen the article by demonstrating the real-world applicability of the discussed methods. Detailed case studies or examples of quantum computing successfully solving specific combinatorial routing problems would provide concrete evidence of the techniques' effectiveness and encourage further exploration in practical settings.
- *Balanced Discussion:* While the hardware focus is essential, a more balanced discussion that also addresses software and algorithmic challenges could provide a more holistic view. Exploring the current limitations and advancements in quantum algorithms, as well as the integration of software with quantum hardware, would offer a comprehensive understanding of the field and its future directions.

7.0 CONCLUSION AND FUTURE WORK

Quantum computing has grown as a significant approach for addressing complex combinatorial optimisation problems, especially in the area of routing. Quantum algorithms, including “Quantum Annealing (QA), Quantum Approximate Optimization Algorithm (QAOA), and Quantum Reinforcement Learning (QRL)”, have shown considerable promise in improving the efficiency of routing solutions relative to classical approaches. Quantum techniques provide a distinct advantage by utilising the unique properties of quantum mechanics, including superposition and entanglement, to investigate problem spaces with greater efficiency. Routing challenges, such as the Traveling Salesman Problem (TSP), Vehicle Routing Problem (VRP), and network routing, have shown improvement through quantum methodologies, indicating the possibility of achieving more efficient solutions in reduced timeframes. The complete realisation of quantum computing capabilities is constrained by existing limitations in quantum hardware, including qubit coherence times, error rates, and qubit connectivity. Hybrid quantum-classical approaches offer a solution that integrates the advantages of quantum computing with classical systems, addressing practical, real-world challenges. As quantum computing technology advances, we anticipate further breakthroughs that will improve the viability of addressing large-scale and complex routing problems through quantum methods. These advancements will be essential in sectors including "logistics, transportation, telecommunications, and healthcare", where routing optimisation is critical.

7.1 Future Work

While the progress made in quantum computing for routing problems is promising, several challenges still need to be addressed to make quantum solutions practical and scalable. Future research can focus on the following key areas:

- *Improving Quantum Hardware:* Improving the stability and power of quantum computers is of the utmost importance. These computers must have more qubits, longer coherence durations, and lower error rates. Improvements in quantum error correction techniques are essential for enhancing the reliability of quantum solutions to routing problems.
- *Algorithmic Advancements:* Research can explore new quantum algorithms tailored to solve routing problems more efficiently. This could involve further optimising existing algorithms, such as QAOA, or developing entirely new quantum algorithms capable of handling dynamic and large-scale routing challenges.
- *Hybrid Quantum-Classical Approaches:* Improving the performance of hybrid models that integrate quantum annealers and gate-based quantum computers with classical optimisation techniques presents a substantial opportunity. Future research may concentrate on developing improved hybrid algorithms that leverage the advantages of both quantum and classical computing.
- *Application-Specific Solutions:* Quantum routing algorithms have many potential uses, but they need more investigation into areas including telecommunications, delivery optimisation, urban traffic management, and transportation logistics. Customising quantum solutions for these areas may result in practical implementations of quantum-based routing optimisation.
- *Scalability:* Enhancing the scalability of quantum algorithms to address the substantial problem sizes encountered in real-world routing issues is a vital area for future research. This involves improving the capacity of quantum algorithms to manage dynamic, time-varying inputs commonly encountered in routing situations, such as fluctuating traffic patterns or varying customer demands.

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